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So Just What *Is* the Academic Language of Mathematics?

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As I mentioned in my last article, there was a time, not so many years ago, when English language learners (ELLs) were routinely placed into math class as their first all-English subject. The belief that ELLs would be able to do math easily because it was "nonverbal" was pervasive. Today, that has changed. Almost all ESL and math teachers now recognize that coming to understand mathematics is a verbal undertaking. Lack of proficiency in the academic language of mathematics is one of the reasons that students who appear to be fluent in English still have difficulty achieving in mathematics.

But exactly what is it that causes the difficulty? Many ESL teachers have a general understanding of the difference between conversational language and academic language. They are familiar with Cummins' distinction between basic interpersonal communicative skills (BICS) and cognitive academic language proficiency (CALP) (Cummins, 1979), and they know that what makes academic language difficult is its increased cognitive complexity and decreased contextual support. But many teachers can't translate that general understanding into the ability to identify specific language difficulties for math, or for any other subject. If you show them a lesson from a textbook and ask them to identify language difficulties, they will pick out a few multisyllabic vocabulary words, ignoring the common multiple-meaning words that can cause more difficulty, or the passive verb constructions, or the connecting words that indicate relationships between parts of a sentence.

While the BICS and CALP distinction can be helpful in a general way, the concept of linguistic register is more useful when thinking about the details of academic language. In linguistics, a register is a variety of a language used for a particular purpose or in a particular social setting. Imagine the conversation of a graduate student relaxing with friends at a local pub on a Saturday night. Now picture the same student during the oral defense of her dissertation on Monday morning. Even if she is talking about her dissertation in both situations, the differences in purpose and social setting will result in very obvious differences in word choice, sentence structure, and discourse patterns.

The register of academic language generally includes features that are used across all academic subjects; each particular subject then has additional features of its own. Features can include pronunciation, intonation, words chosen or not chosen, particular meanings of words, preferred sentence structures, accepted discourse patterns, common ways of accomplishing functions of language, and pragmatic rules. This article will take a closer look at some of the features of the register of academic math language that cause particular difficulty for English language learners.

Vocabulary

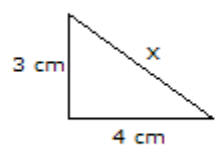
The most obvious difference between other registers and the language of math is its vocabulary. And vocabulary is the first thing many people think of when they focus on language difficulties that ELLs have in math class. It's true that math has many everyday words that have specific mathematical meanings. But math also has many complex, conceptually dense terms, and sometimes a math term incorporates both difficulties. Suppose an ELL has some familiarity with the everyday meanings of the three words in the term *least common multiple*, and tries to use that knowledge to define the term. She might come up with something like *smallest frequent multiplication*. That won't help very much. Even if she knows the math meanings of the three words, she still must combine them into a meaningful new concept, a very challenging cognitive task. It is extremely difficult for an English language learner to combine the meanings of *smallest, shared by two or more things,* and *the product of a particular number and any other counting number* and come up with a meaningful definition.

At this point you might be thinking that, yes, definitions of complex math terms are difficult to understand. However, good math teachers teach vocabulary using concrete examples, not definitions. Here's a concrete example:

> Multiples of 4: 4, 8, **12**, 16, 20, 24, 28, 32, 36
> Multiples of 6: 6, **12**, 18, 24, 30, 36, 42, 48, 54

Does this help our ELL student define *least common multiple*? It may help her gain a nonverbal understanding of the concept, but what does she do when she finds the question *Explain what a least common multiple is* on a math test?

That was an obvious example. *Least common multiple* is the kind of thing that should be recognized as a source of potential difficulty for ELLs, even by teachers without ESL training. The more insidious difficulties are those that even teachers with ESL training don't recognize unless they analyze the kinds of mistakes their students make. Here's an example (from Moschkovich, 2007). A test question shows this figure and says: *Find x*.



The student writes *Here it is*, and draws an arrow to the *x*.

Other vocabulary problems are even more difficult to identify because they involve comprehension, which is not observable. For example, it was only a think-aloud protocol used in a research study that allowed Celedón-Pattichis (2003) to discover why one Spanish-speaking student did not correctly solve a word problem. The problem was:

> On Saturday, 203 children came to the swimming pool. On Sunday, 128 children came. How many more children came to the pool on Saturday than on Sunday?

Faced with an incorrect answer to this problem, most teachers would assume that the *how many more* construction was the cause of the difficulty. In this case, however, the think-aloud revealed that the student had misinterpreted *than* as *then*. If the student thinks that the question is asking how many of the students who came to the pool on Saturday then also came on Sunday, the facts given in the problem are not sufficient to answer the perceived question.

And you thought that vocabulary was the "easy" language difficulty to identify and remediate!

Semantax

My next example, comparisons, is hard to categorize. It's based partly on word meanings (vocabulary? lexicon? semantics?), partly on what we used to call just plain "grammar" (which is actually mostly morphology), and partly on sentence structure (syntax). I've called it *semantax* (a term that is used more and more frequently in linguistics to refer to the interrelationship of semantics and syntax).

At this point, you may be wondering why I chose comparisons. Aren't comparisons relatively easy?

You just add *-er* or *-est* to the end of the word. Or use *more/most*, or *less/least*, or *as* ___ *as*, or ... . All right, maybe they're a little more complex than they seem. And don't forget that when you're comparing two things, you need to use than, which Spanish speakers will often produce as *that*. None of this is the real problem, however. Consider the sentence *Israel is 3 years older than his brother*. It seems pretty straightforward. In order to understand this sentence, however, students have to understand the concept of comparison. But comparison is not a thing, it's a relationship. Understanding a concept is difficult when the concept consists of a relationship between two other concepts. In this example, ELLs have to think about one thing, think about the other, and think about the relationship between the two things. That's a lot harder than learning the concept of *digit*, for example, which is nothing more than *0, 1, 2, 3, 4, 5, 6, 7, 8, 9*.

My next examples deal with what would usually be called syntax, or sentence structure. However, I'm going to include them in my semantax category, because it's not the sentence structure in itself that causes the difficulty, it's how the sentence structure affects meaning.

Passive verbs: Passive verbs normally cause more difficulty for speaking and writing than they do for understanding, but when they are combined with other difficult aspects of language, they can make what might have been an easy question totally incomprehensible for ELLs. Cases in point:

- Which number is represented by the shaded part of the figure? (a drawing shows a circle with 6 parts; 4 are shaded) An ELL student thinks: *No number is by the shaded part of the figure*.
- Which is read one million, five thousand, seventy-nine? (the question is followed by 4 choices, one of which is 1,005,079) An ELL student wonders: *Which what are they talking about? Do they want to know which person is reading this number*?
- How much change should she have received? (at the end of a word problem about buying something and getting the wrong change) An ELL student asks: *Does this mean how much change she has, or how much change she gets*?

The people who write these questions (in this case, item writers for a fourth-grade math test in a state that shall remain nameless) should have computers with one of those grammar checkers that waves a red flag whenever a passive verb is used (oops-red flag!).

If clauses:

- If you multiply 8 times 5, the answer is 40.
- If I take one counter out of the bag, the probability that it will be red is 1 in 2 (assuming there are 3 red counters, 2 blue counters, and 1 green counter in the bag).
- If we had cut the pizza into 8 pieces instead of 4, the pieces would have been smaller.

If you think that the word *if* in each of those sentences means the same thing, you are mistaken. But nobody teaches ELLs about different meanings of *if*. We just teach them that *if* means something that didn't really happen. If you can describe the differences among these three *if* sentences, you get a star. And if you can create interesting, interactive lessons to teach these three different *if* constructions to ELLs, you get five stars.

Prepositions:

- 6 divided *by* 12 is 1/2 (or 0.5).
- 6 divided *into* 12 is 2.

Why doesn't *6 divided into 12* mean the same thing as *6 divided by 12*? If you divide 6 into 12 equal groups, you get 1/2 in each group.

- 6 multiplied *by 12* is 72. I multiplied.

- 6 exceeds 12 by 6. I subtracted.

This is one reason why we try not to teach ELLs to use so-called "clue words" to decide what operation to use.

Lack of correspondence between symbols and the words they represent:

- In a dictation, students are asked to write the division problem *648 divided by 8*. If they write it *648 ÷ 8*, they will have no problems. If, however, thinking that 648 is a large number, they choose to write it as a long division problem, they will very likely write a problem that is read as *8 divided by 648*. It takes ELLs a long time and a lot of practice to learn that they can't write the symbols in the same order in which they hear them.

- In an algebra problem, students read the phrase *the number* x *is 10 less than the number* y. They know they have to write an equation in order to solve the problem. So they write *x* ("the number *x*") = ("is") 10 - ("10 less than") *y* ("the number *y*"). They never realize that the resulting equation, $x = 10 - y$, should really be $x = y - 10$. They need still more practice to learn that they can't write the symbols in the same order in which they hear them.

Looooooooong, complex sentences: Bielenberg and Fillmore (2004/2005) provide an example of how mathematical discourse piles phrases on top of clauses on top of more phrases and clauses, until ELLs need a roadmap to find their way around. In a problem about how many 7- to 10-minute speeches could be given in a 2-hour class (from a sixth-grade math test in another unnamed state), students are asked:

- Which of the following is the *best* estimate for the total number of student speeches that could be given in a 2-hour class?

As the authors point out, this question contains a complex noun phrase, which contains a complex prepositional phrase, which contains a relative clause construction, which contains a passive construction.

Is this kind of language really necessary? Evidently, the test writers think it is, and until we can change their minds, we have to teach our ELLs how to understand it.

Discourse Patterns

Everybody knows about genres in literature. We teach students to write fictional narratives, persuasive essays, and expository presentations. They know how to recognize personal letters, poems, plays, perhaps even blogs, and most certainly IMs. But we don't teach them anything about the genre of math word problems. So they approach "story problems" in they same way they would approach a story. In a story, if they don't know a word, they can often figure it out from the context; they can look at the pictures to help get an idea of what the story is about; thinking about things they know about in the story will help them understand. None of that works with mathematical word problems. Word problems are usually short (on purpose, to leave more room on the page for more problems or other things). Because they are short, they lack any kind of real context that might help students understand them, and they lack the natural redundancy of language that helps make it comprehensible. The context of a word problem is an artificial one, invented in order to provide students an opportunity to use math to solve "real" problems. So the pictures usually have nothing to do with the mathematics of the problem. And this artificial context makes it unlikely that anything a student might know about the people or things used it the problem will be helpful for understanding it. Add to this the fact that the language used in the problem is often more complex than it needs to be, and a problem that appears very simple to those of us who are familiar with the genre of word problems becomes incomprehensible for ELLs.

When ELLs (and native speakers of English) try to apply reading comprehension strategies that are successful in other genres, they experience failure. This leads them to develop other strategies that they perceive as more successful with word problems. They look for clue words, or choose an operation based on the absolute or relative size or the numbers. Wiest (2003) points out that these strategies often work because word problems have such a "typical" format. They are a genre unto themselves.

So let's stop pretending that word problems are real, and that if students can't solve them, the problem must be the math. Let's acknowledge the reality that word problems are "stylized representations of hypothetical experiences, not slices of everyday existence," written in the register of "word-problemese" (Lave, 1993, quoted in Wiest, 2003). And let's teach our students the strategies they need in order to understand them, along with the strategies they need in order to solve them.

Background Knowledge

Here's another example from Bielenberg and Fillmore (2004/2005) that shows how lack of background knowledge can prevent ELLs from solving a seemingly simple problem:

- A submarine is 285 feet under the surface of the ocean. A helicopter is flying at 4,500 feet above sea level. Given that the helicopter is directly above the submarine, how far apart are they?

At first glance, this problem does not appear to present difficulties. There is no technical math vocabulary, and most ELLs learn words such as *submarine* and *helicopter* fairly easily, especially if they speak a first language where the words are cognates with English.

The authors' analysis, however, identifies several areas where incomplete background knowledge could prevent students from arriving at the correct answer:

- *sea* and *ocean* are used as synonyms; *sea level* and *surface of the ocean* refer to the same level;
- the term *sea level* is an abstraction; it can apply to places that are nowhere near the ocean, and places that are below sea level are not literally under water;
- the phrase *how far apart* does not specify horizontal distance or vertical distance; students must interpret the information in *under the surface of the ocean* and *above sea level* in order to determine this.

In addition to these issues of background knowledge, there are also language difficulties:

- the phrase *given* that is crucial for correctly understanding the problem, but is not something that ELLs are likely to understand;
- references of pronouns can be difficult, particularly *they* because it can refer to either people or things.

Conclusion

Since the academic language of mathematics (another red flag!) presents so many difficulties for ELLs, what can be done? There is one simple answer: Teach it. We can't wait for students to "pick it up" on their own. Not even native speakers pick up academic English on their own; it has to be taught, explicitly and comprehensively.

One of the conclusions arrived at by Francis et al. (2006) after an extensive review of the literature on teaching math to ELLs says it more formally:

> Academic language is as central to mathematics as it is to other academic areas. It is a significant source of difficulty for many ELLs who struggle with mathematics. ... [T]he oral and written language of mathematics—or the mathematics *register*—should be ... explicitly integrated into the curriculum. (pp. 37-38)

The next article in this series on mathematics and ELLs will provide a detailed "how-to" for selecting, teaching, and assessing the academic language of math.

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