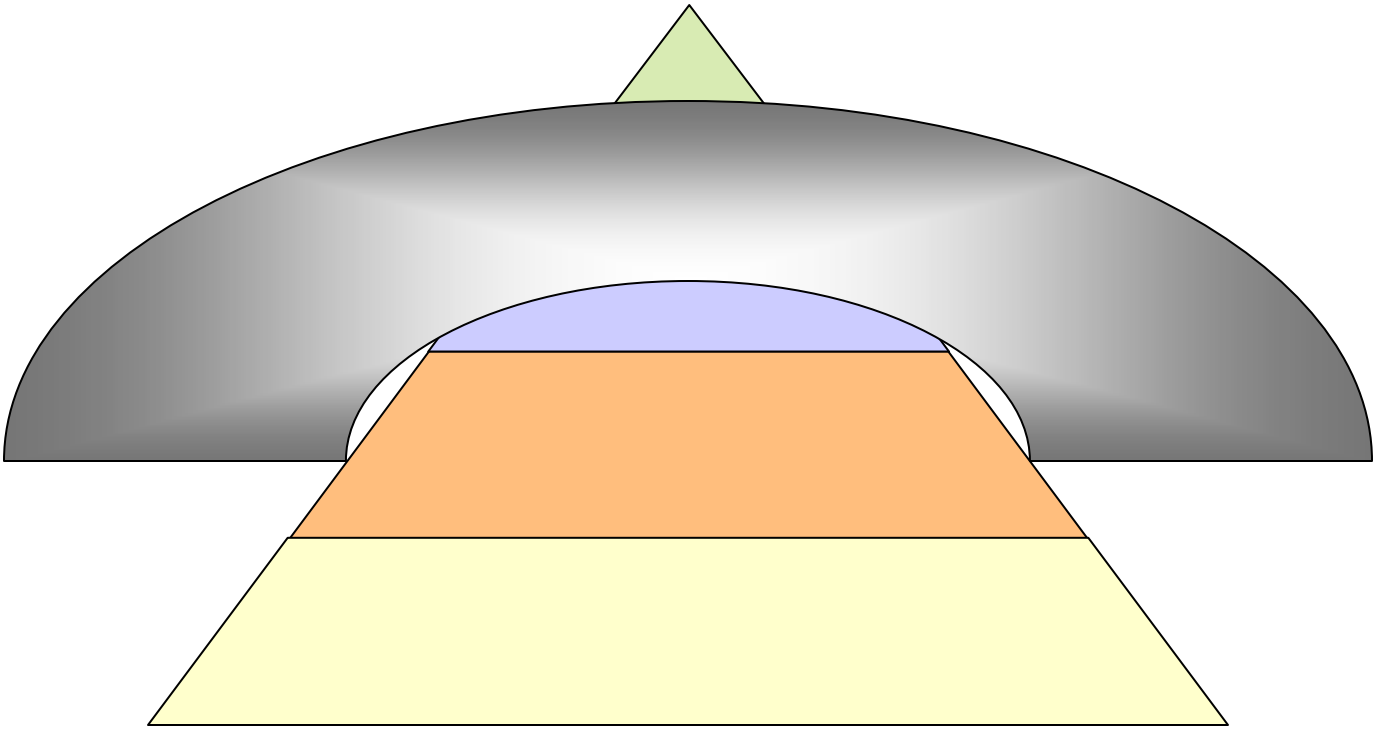
Building a Bridge to Academic Vocabulary in Mathematics



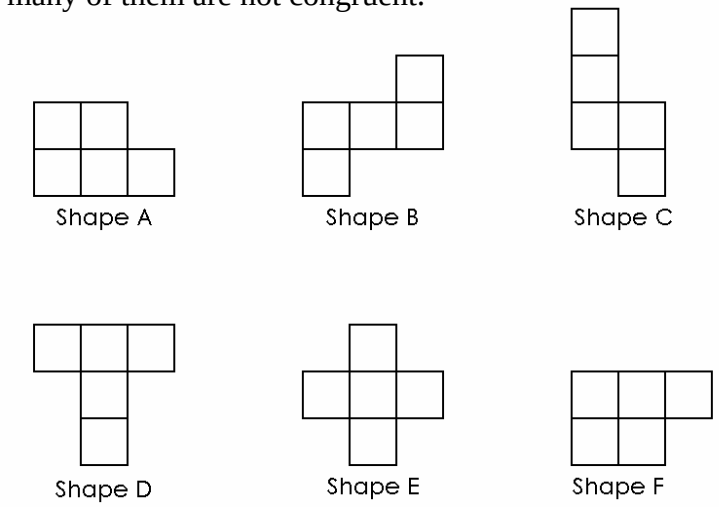
AISD Elementary Mathematics Department

How Students Develop a Repertoire of Academic English in Mathematics

Developed and researched by the AISD Elementary Mathematics Department

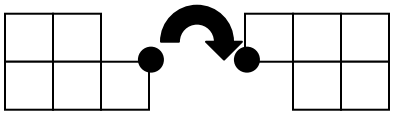
Students do not learn mathematical vocabulary by memorizing definitions. Rather, they construct meaning for mathematical vocabulary by actually doing authentic and meaningful mathematics. Nevertheless, it is unrealistic for teachers to expect that all students will somehow absorb targeted vocabulary by simply engaging in mathematical investigations. The teacher must be purposeful in constructing learning experiences that direct the student's attention to specific vocabulary. Consider the following episode observed in an AISD 3rd grade classroom in which the students are working with geometrical concepts and vocabulary focusing on congruency and transformations:

Students in Mrs. Villareal's third grade class at Brown Elementary were working on an end-of-unit assessment task from the Investigations unit, *Flip, Turns, and Area*. The students were asked to look at the six arrangements of five squares shown below and determine how many of them are not congruent.

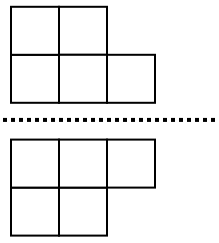


One student wrote the following explanation for why **Shape A** and **Shape F** are congruent figures: "They're the same but this one (*an arrow is drawn to connect Shape A to Shape F*) is just turned over." Notice how this explanation, although it makes sense, is not altogether accurate. The student is right in that the two figures are indeed the same. However, if Shape A is turned, it will not look identical to Shape F. Instead, Shape A has to be flipped over and then turned so that it will have the same orientation as Shape F.

Example of what Shape A ***would look like if it were "turned"***



Example of what Shape A ***would look like if it were "reflected"***



In addition to his conceptual error in describing the correct motion that would make these two figures' orientation identical, this student's failure to connect the formal mathematical term—*rotation*—to describe how **Shape A** was transformed to match **Shape F's** orientation illustrates how students' understanding of word meanings is often incomplete. Although Ms. Villareal had already introduced the word ***rotation*** to her class, this student had obviously not made a complete connection to this terminology as a way to label the corresponding application or "turning" motion that positions a figure in space.

This classroom episode shows the importance of exposing students to multiple ways to represent their understanding of mathematical concepts and formal vocabulary. For example, Ms. Villareal could have required her students to use the new terminology—*rotation*, *reflection*, and *translation*—in their math journals as a way to reflect upon and describe what they did that day during math time. This, however, is only one instructional strategy to further deepen the student's understanding of transformational geometric vocabulary. As with all types of learning, students need continuous opportunities that are purposefully and strategically orchestrated by the teacher to revisit and target mathematical vocabulary and concepts.

What teachers should be foremost concerned about is not only ***if*** their students understand the meaning of a new mathematical term, but also ***how*** they understand it. In his book, *Elementary and Middle School Mathematics, Teaching Developmentally*, John Van de Walle promotes a "constructivist" theory that clearly explains how students should learn mathematics:

> ***"Understanding is a measure of the quality and quantity of connections that a new idea has with existing ideas. The greater the number of connections to a network of ideas, the better the understanding."***

The AISD Elementary Mathematics Department suggests that teachers follow Van de Walle's constructivist theory when teaching mathematical vocabulary to students. We recognize that there is no prescribed format or 'magical' formula to ensure that our students in AISD will understand and use academic mathematical English. Each student will have a unique understanding of math vocabulary, ranging in various levels of complexity and sophistication as he/she continues to grow as a mathematical thinker. What this department does recommend is that teachers use a multi-representational approach to teach math vocabulary so that our students will have a stronger foundation and understanding of formal mathematical terminology before entering Middle School. This simply means that students must show and apply their learning within the context of real mathematical experiences that offer many opportunities to revisit, apply, and show evidence of understanding using a wide range of strategies and representations—drawing, identifying, classifying, describing, and explaining formal mathematical vocabulary.

Through active discourse and interaction with important mathematical ideas, the teacher must strategically plan learning experiences that purposefully bridge their students' informal language to formal mathematical vocabulary. The remainder of this document provides instructional strategies that will help teachers construct this bridge for future student success with academic English in mathematics.

Informal vs. Formal Language and Vocabulary

The table below lists some examples of informal language used by students to describe mathematical ideas. The proceeding column lists the corresponding formal mathematical vocabulary:

Informal Vocabulary	Formal Mathematical Vocabulary
Split	Divide
Take away	Subtract
Slide	Translation
Turn	Rotation
Flip	Reflection
Change	Transformation
Same	Congruent/Equal
More	Greater
Less	Fewer
Holds	Capacity
Inside	Interior/ Area
Around	Perimeter
Cross	Intersection
Total	Sum

5 Components for Scaffolding Formal Mathematical Vocabulary

The AISD Elementary Mathematics Department lists 5 components for scaffolding students' understanding and use of informal vocabulary with formal mathematical vocabulary:

- **Modeling**—The teacher uses the targeted vocabulary within a meaningful context, often "revoicing" the student's informal language with the formal mathematical terminology.

  Example: **Student**—*"I split 14 and got 7."* **Teacher—***"So you mean that 7 is half of 14."* **Student—***"Yes, two 7's make 14."* **Teacher—***"Yes, you can divide 14 by 2 to get a quotient of 7."*

- **Discourse**—The teacher orchestrates activities and poses questions that elicit student talk about mathematical ideas as they arise within the course of the lesson.

  Example: The students are making arrangements with 5 color tiles. The teacher makes two arrangements on the overhead and asks: *"Are these two pentominoes congruent? How do you know? What do I mean by congruent? Take 5 minutes and talk with your partner."*

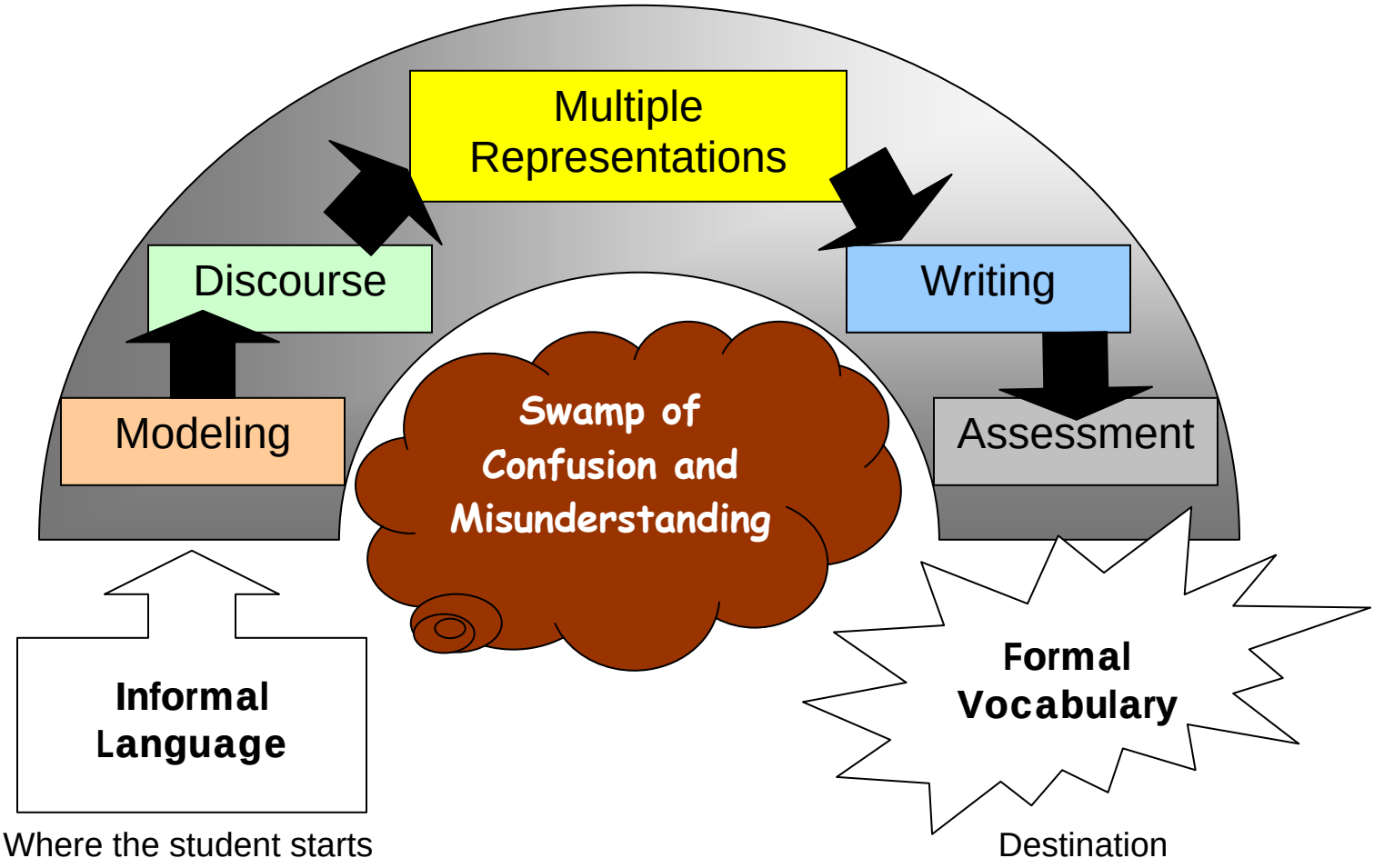
- **Multiple Representations**—The teacher uses various graphic organizers and displays—e.g. ***Venn Diagrams***, ***concept maps***, ***Frayer Models***, ***tables***, ***word banks***/ ***banks***, etc.—to reinforce targeted vocabulary.

- **Writing—**The students use the targeted vocabulary to reflect on and organize their thoughts around related mathematical ideas—e.g. ***journals***, ***justification of solution strategies***, etc.

- **Assessment**—The teacher uses informal observation and formal assessment tools to determine the depth to which the student understands and explains his/her understanding using the targeted vocabulary.

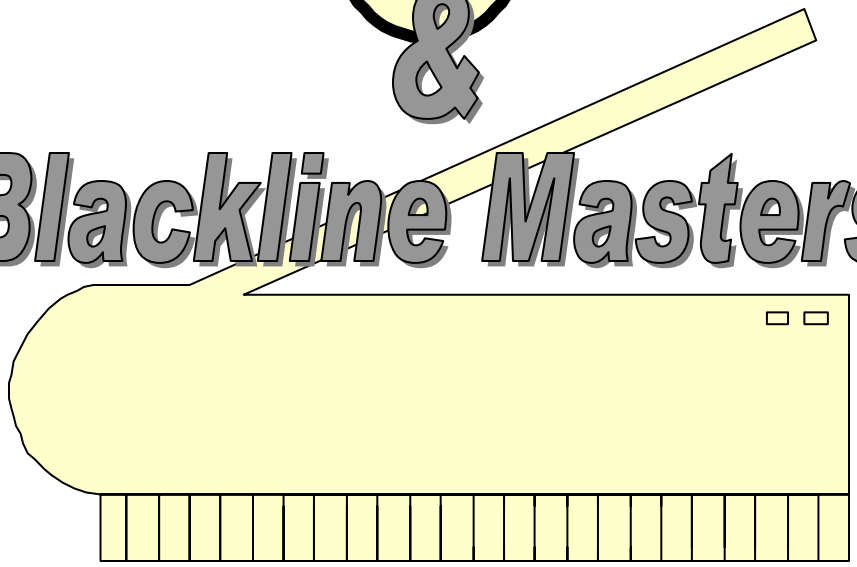
Building The Bridge:

An AISD Model for Scaffolding Students' Informal Language With Formal Mathematical Vocabulary



****Note that these 5 components for scaffolding formal mathematical vocabulary are not necessarily sequential. Instead, they are recursive and ongoing.**

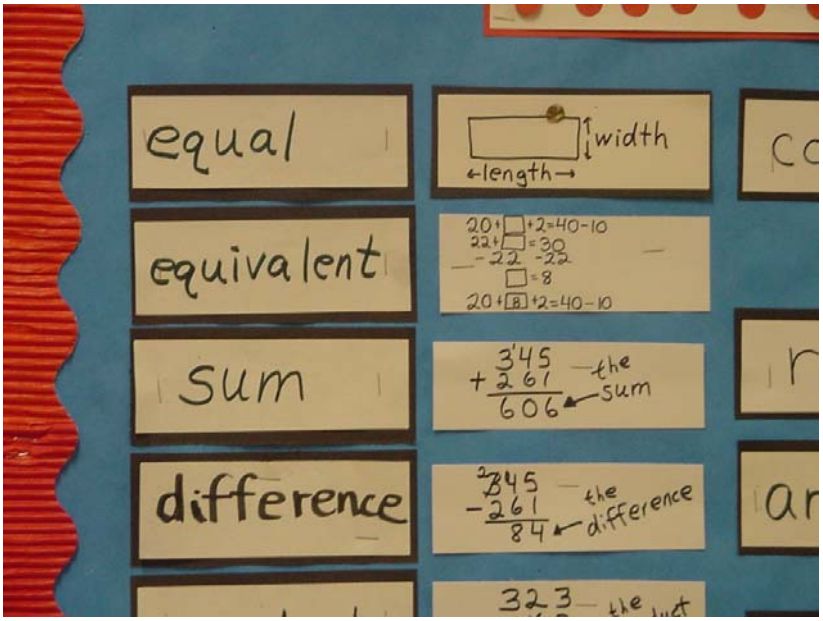
Instructional Strategies & Blackline Masters



Word Bank

One way to reinforce new vocabulary is to add words to a "word bank" in the classroom. This is simply a visible location in the classroom for posting words where students can see them. It is helpful to post a student-created definition and an example next to the term. Adding new vocabulary to the word bank gives it a constant presence in the classroom. Children see the written word, use the word orally, and, when they are ready, copy and write the word when they are explaining their ideas in writing.

We need to make many decisions as we provide opportunities for students to develop vocabulary and students need a wide variety of experiences in order to have a deep understanding of the mathematical concept that is expressed in mathematical words. For example, we might think that adding color makes a shape more interesting, young children may associate color as an attribute of the shape. In other words, if a triangle is green, young children may develop the misconception that all green shapes are triangles. Orientation of shapes is another common misconception. Multiple student-created examples, and sometimes non-examples, will address these issues.



Defining/Describing Terms Using Word Banks

Asking students to define or describe a mathematical term using words from a word bank helps them make connections between math vocabulary words. Below is an example of what this activity would look like if the directions were written on the board or on the overhead. Use words from the related unit of study or from the Word bank. ***(The example below is not grade level specific.)***

Define the term *fraction* using these words from the Word Bank:

Word Bank
whole
numerator
denominator

Define the term *pentagon* using these words from the Word Bank:

Word Bank
polygon
sides
angles

Define the term *cube* using these words from the Word Bank:

Word Bank
faces
edges
vertices

Define the term *circle* using these words from the Word Bank:

Word Bank
chord
diameter
radius
center

Describe a _____ using the words in the Word Bank.

Defining/Describing Terms Using Word Banks (Black line)

Define the term _____ using these words from the Word Bank:

Word Bank

Define the term _____ using these words from the Word Bank:

Word Bank

Define the term _____ using these words from the Word Bank:

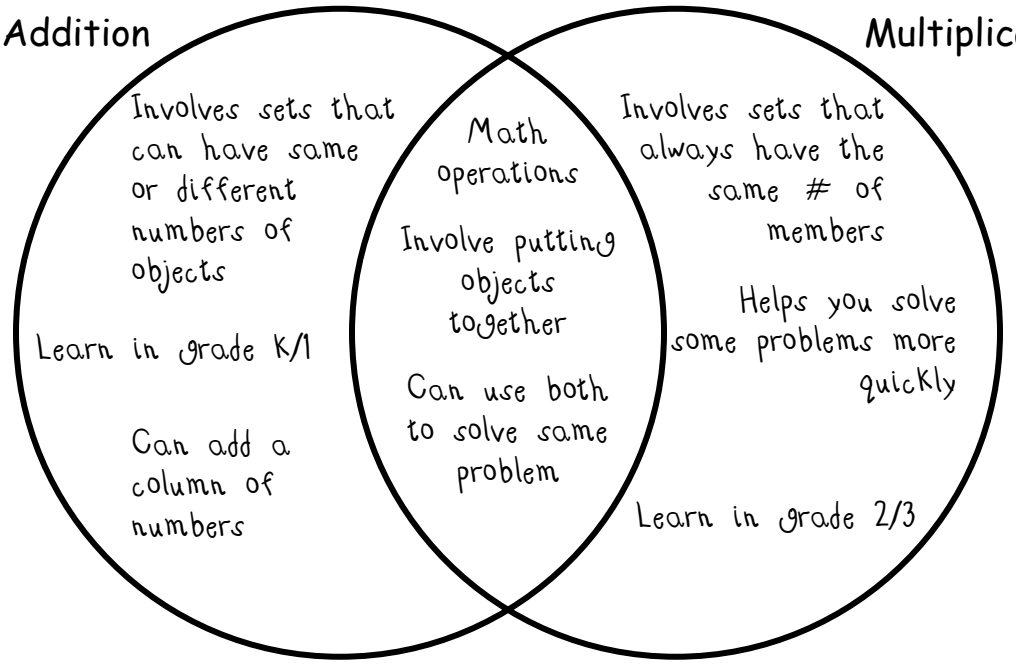
Word Bank

Define the term _____ using these words from the Word Bank:

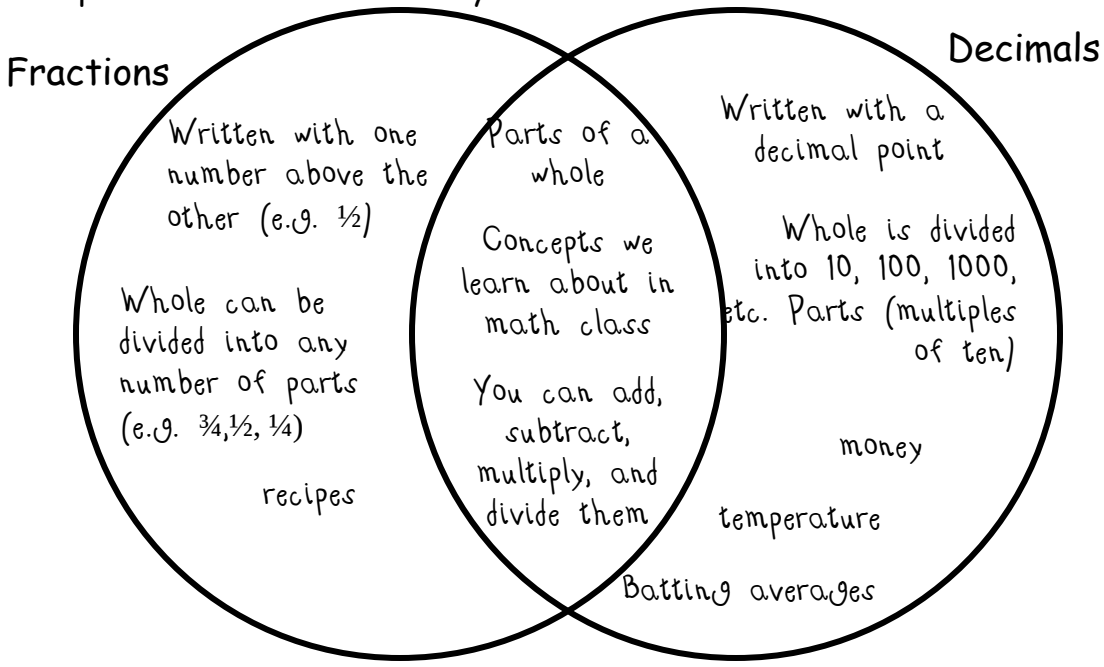
Word Bank

Describe a _____ using the words in the Word Bank.

Classifying Terms Using the Word Bank & Venn Diagrams

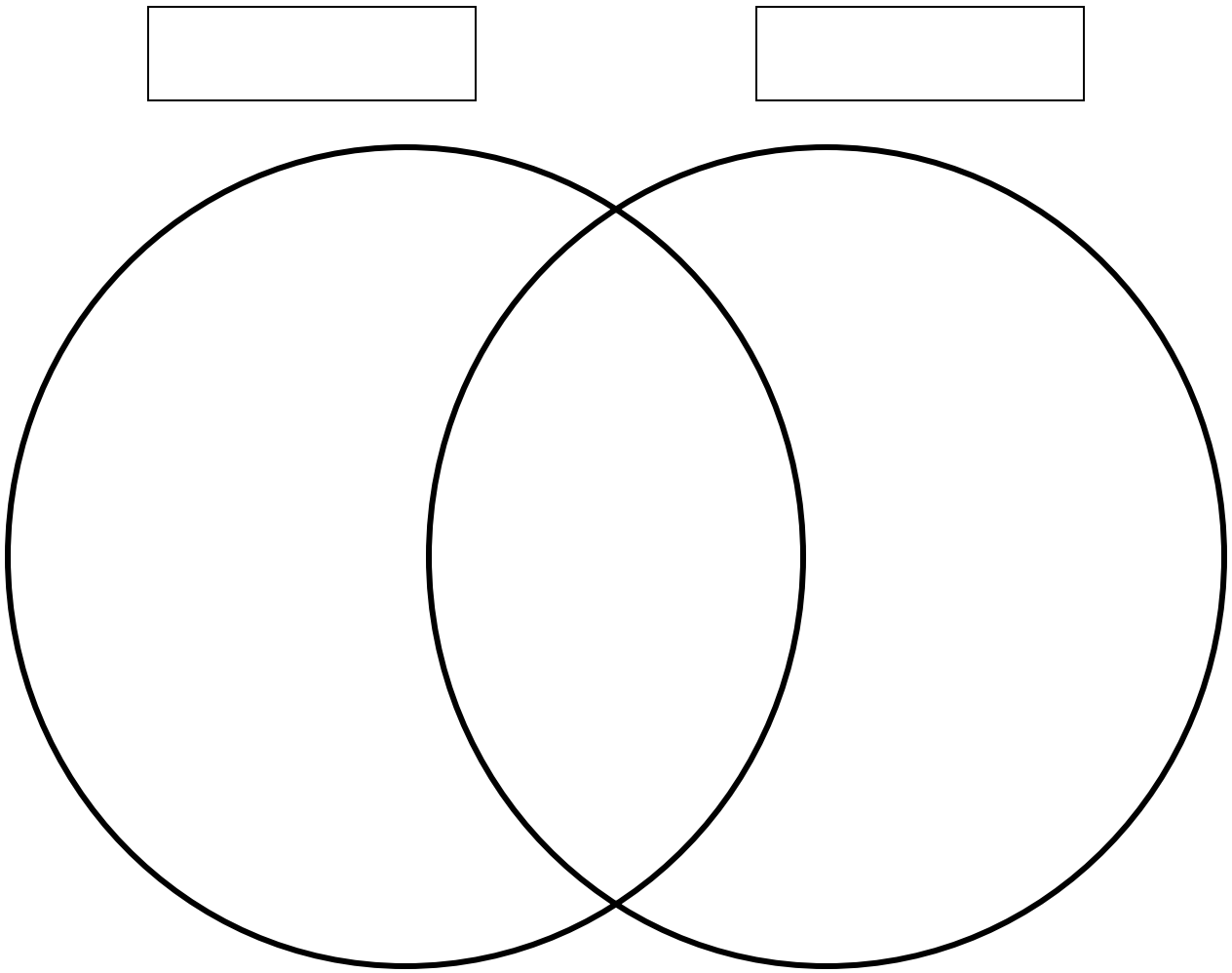


Extension: Write a story problem that can be solved using addition or multiplication. Solve it both ways



Extension: Explain the difference between a fraction and a decimal.

Classifying Terms Using the Word Bank & Venn Diagrams (Black Line)



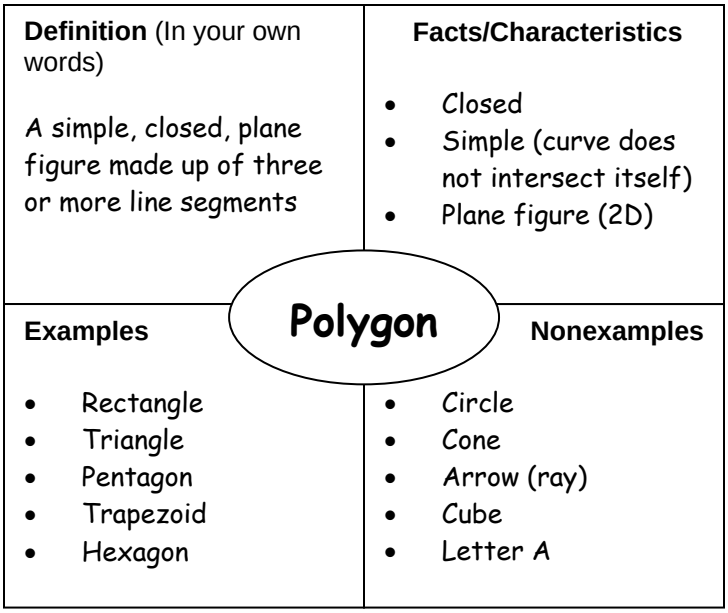
Extension:

Frayer Model

The Frayer Model is a word categorization activity that helps learners develop their understanding of concepts. Students provide a definition, list characteristics or facts, and provide examples and nonexamples of the concept. There are many concepts in mathematics that can be confusing because of their close relationships (e.g. prime numbers and factors) and/ or their specialized features (e.g. functions). This strategy provides students with the opportunity to understand what a concept is and what it is not. It gives students and opportunity to communicate their understanding and to make connections by providing examples and nonexamples from their own experiences with the concept.

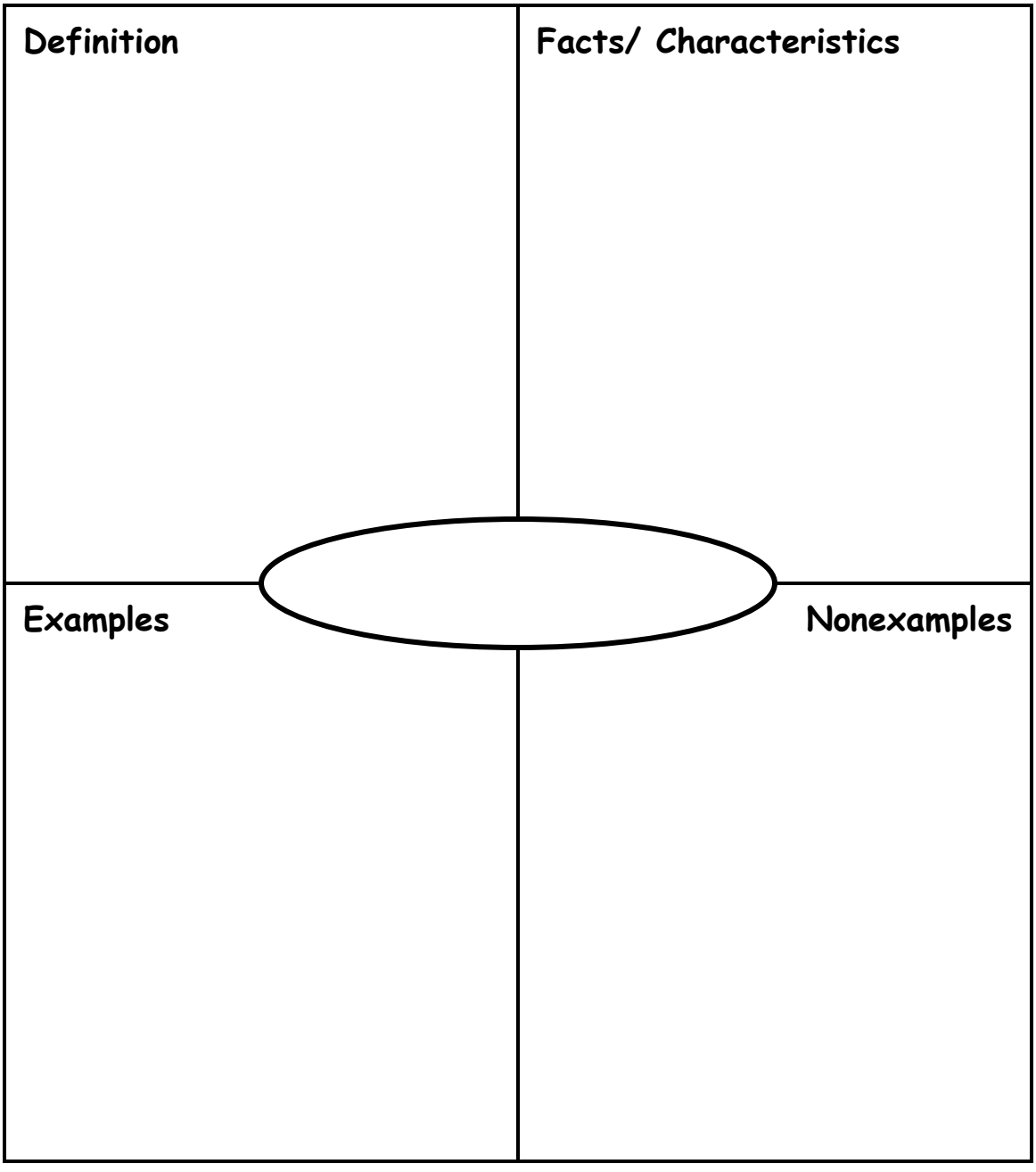
How to use it:

1. Take a concept that might be confusing because of its relational qualities.
2. Explain the Frayer model diagram.
3. Model how to fill out the diagram by asking for items to list in the "examples" section. Then ask for nonexamples. By referring to the examples & nonexamples, ask students what facts or characteristics might be given about the term. Finally, ask students to use all the information discussed to come up with a group definition of the term.
4. When students are familiar with the diagram, let them practice with assigned terms.
5. Once their diagrams are completed, let the students share their work with other students. Display students' diagrams as posters throughout the unit so students can refer to the words and continue to add ideas.



Barton, M.L., & Heidema, C. (2002) Teaching Reading in Mathematics (2nd ed.). Colorado: Mc Rel

Frayer Model (Black line)



ABC Brainstorming

What is ABC Brainstorming?
Before having your students talk about a major topic, it is essential to activate their background knowledge about it. One way to do this is the ABC Brainstorm. The idea is meant to be fairly simple. Students try to think of a word or phrase associated with the topic, matched to each letter of the alphabet.

How Does It Work?
Use that alphabet chart below or have students list the letters of the alphabet down a sheet of paper. Let them work individually at first, thinking of as many words as they can that could be associated with the topic you identify. Note: The topic should be big and general enough that students can actually think of a lot of possible terms. Then, in no particular order, let them begin filling in the blanks beside the letters of the alphabet.

It seems to work well if you give student enough time to think of a lot if ideas but then let them pair up or work in small groups to share ideas. In this way, you can let the brainstorming function like a Think-Pair-Share. Then, let students share possible terms for the different letters of the alphabet as you record their responses on a large chart or chart paper. Be open to a wide range of possibilities! Make sure students know that you're not looking for exact answers, just justifiable and relevant ones.

What sorts of Topics Are Good for an ABC Brainstorm?
Topics you are introducing so students have an anchor, or it can be a good recap for topics already studied.

ABC Brainstorm Strategy

Title: _____

A	**B**	**C**	**D**
E	**F**	**G**	**H**
I	**J**	**K**	**L**
M	**N**	**O**	**P**
Q	**R**	**S**	**T**
U	**V**	**W**	**XYZ**

Cue Cards

In this game, students match mathematics expressions to the verbal language used to read them. This game could be used in a mathematics class as a review or as a novel way to practice with symbol vocabulary. As students hear the expressions read aloud, they construct meaning and validate their understanding of symbols.

How to use it

1. Provide each student (or pairs of students with a set of six to eight cards. Each card should have a mathematics statement given in symbols.
2. Prepare cue cards with phrases to read for the mathematics statement. You may have more than one cue card corresponding to one statement.
3. Display and read the cue cards aloud one at a time. As each phrase is presented, ask students to select and hold up a matching response card. Visually check for correct responses.

Set of cards for each student:

$n + 7$	$7 - n$	$n/7$	$7/n$
$7n$	$n > 7$	$n < 7$	$n - 7$

Cue Cards:

A number n is less than 7	7 more than a number n	7 divided by a number n	The quotient of a number n and 7
A number n decreased by 7	The sum of a number n and 7	7 subtracted from a number n	7 less than a number n
7 is less than a number n	The difference between n and 7	The product of a number n and 7	A number n increased by 7
	7 times a number n	7 is greater than a number n	

Barton, M.L., & Heidema, C. (2002) Teaching Reading in Mathematics (2nd ed.). Colorado: Mc Rel

Cue Cards (Black line)

$n + 7$	$7 - n$
$n - 7$	$7n$
$n/7$	$7/n$
$n > 7$	$n < 7$

Cue Cards (Black line)

A number ***n*** is less than 7	7 more than a number ***n***
A number ***n*** decreased by 7	The sum of a number ***n*** and 7
7 is less than a number ***n***	The difference between ***n*** and 7
7 divided by a number ***n***	The quotient of a number ***n*** and 7
7 subtracted from a number ***n***	7 less than a number ***n***
The product of a number ***n*** and 7	A number ***n*** increased by 7
7 times a number ***n***	7 is greater than a number ***n***

Concept Circles

A versatile categorization strategy for students to study words or terms critically, relating them conceptually to one another. With a concept circle, students identify common attributes or a concept relationship that exists among several terms. As a pre-reading activity, concept circles can involve students in predicting and discovering relationships, beginning the process of defining a concept. As a reinforcement or extension activity, students identify a concept (perhaps more than one) that relates several terms, terms, thinking about common attributes and extensions.

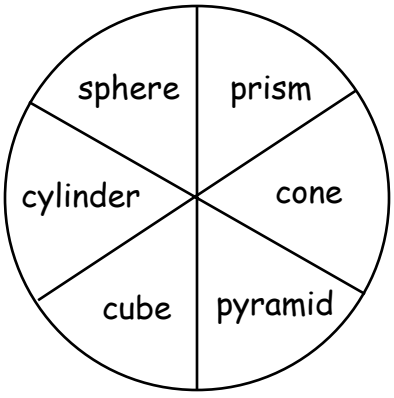
How to use it:

1. Choose common attributes or relationships among a number of terms.
2. Draw a circle divided into sections (three to six) and put a term (word or phrase) into each section, all of which have the identified attributes or are related in a similar way.
3. Direct students to identify the common attributes or name the relationship that exists among the terms in all sections of the circle.

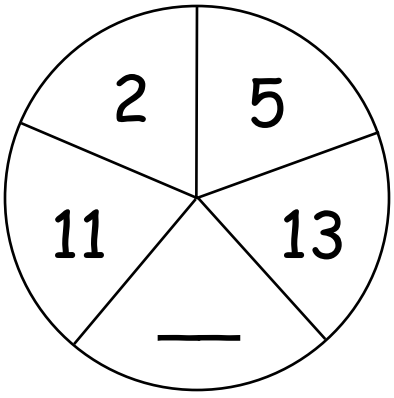
Modifications:

1. Leave one section of the circle empty and direct students to identify the concept relating all the terms given in the other sections and to fill the empty section with a new term that also fits with this concept.
2. Choose a term for just one section of the circle that is not an example of a concept and terms for all the other sections. Then direct the students to find the term that does not belong and identify the concept that relates the other terms.

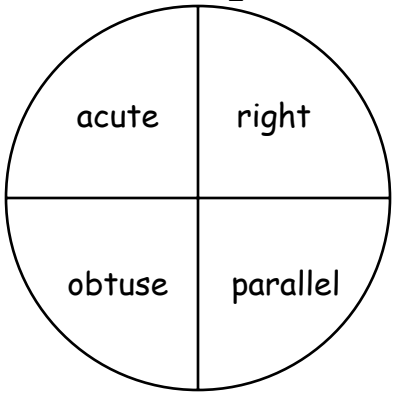
Concept: Solid Figures



Concept: Prime Numbers

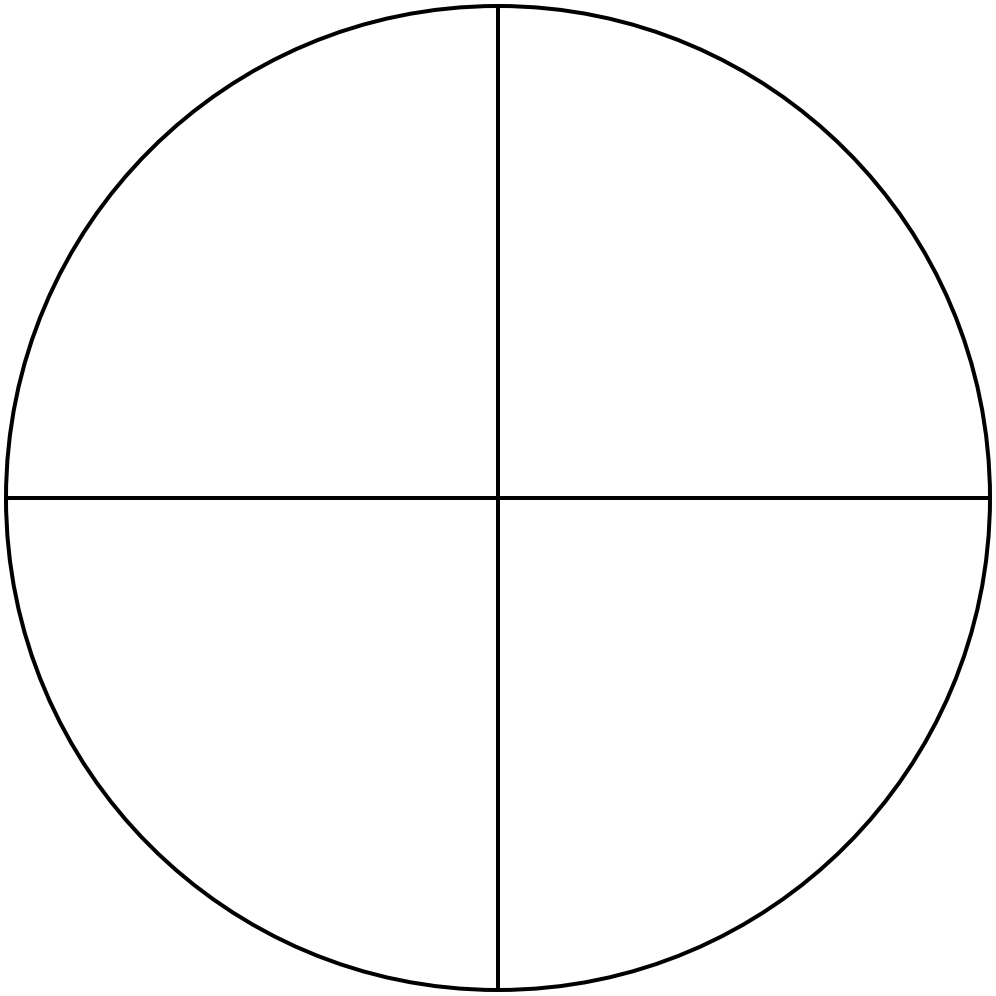


Which term does not belong?



Concept Circles (Black Line)

Concept: _____



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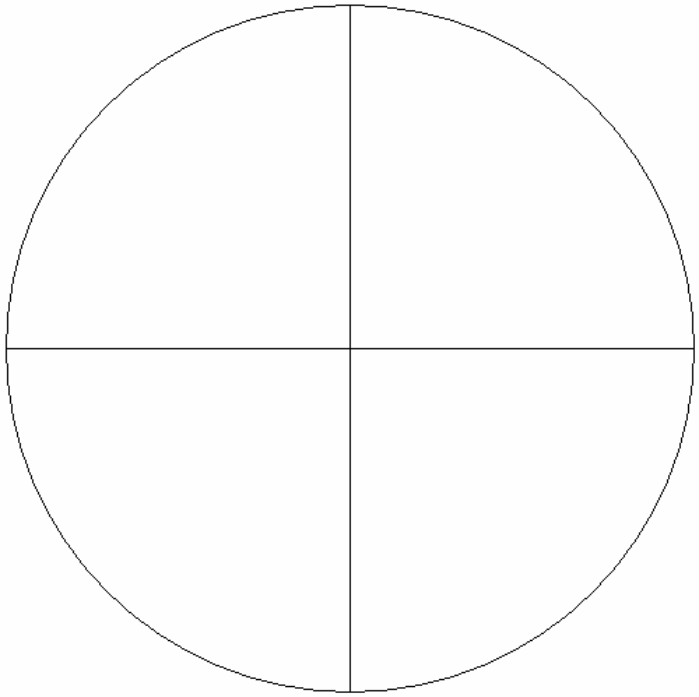

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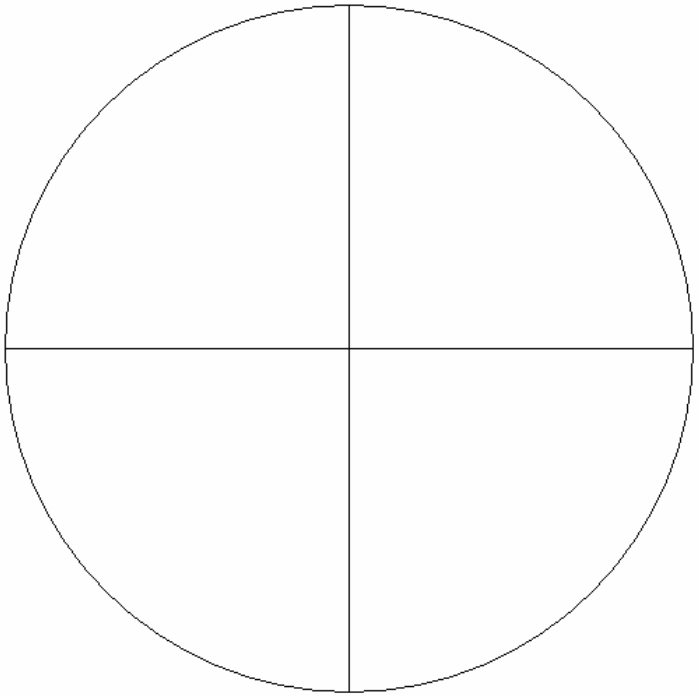

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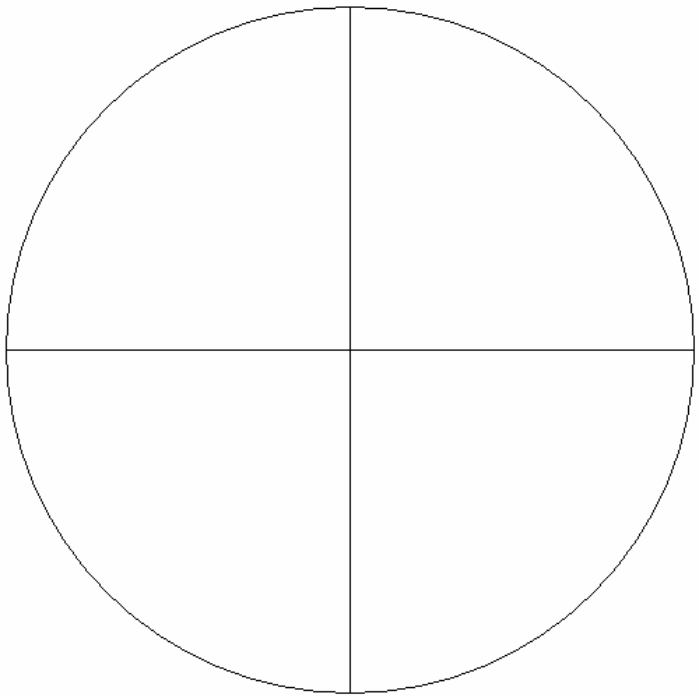

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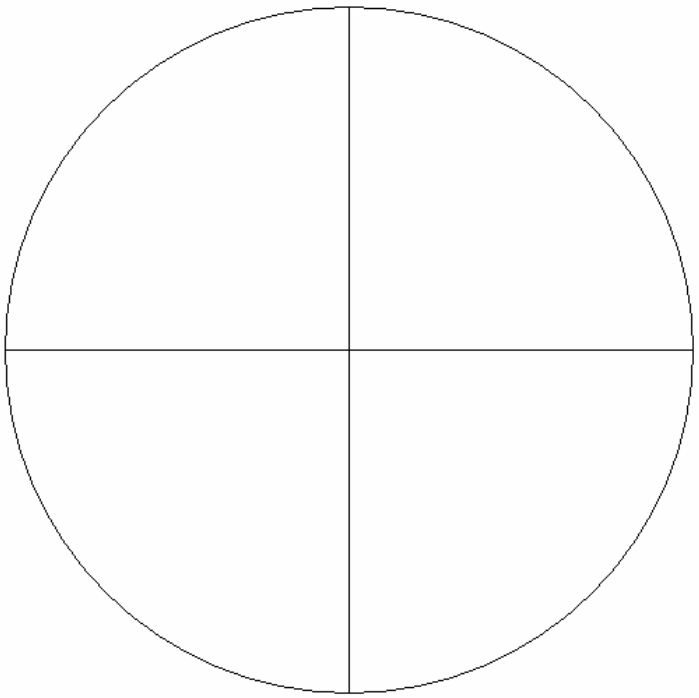

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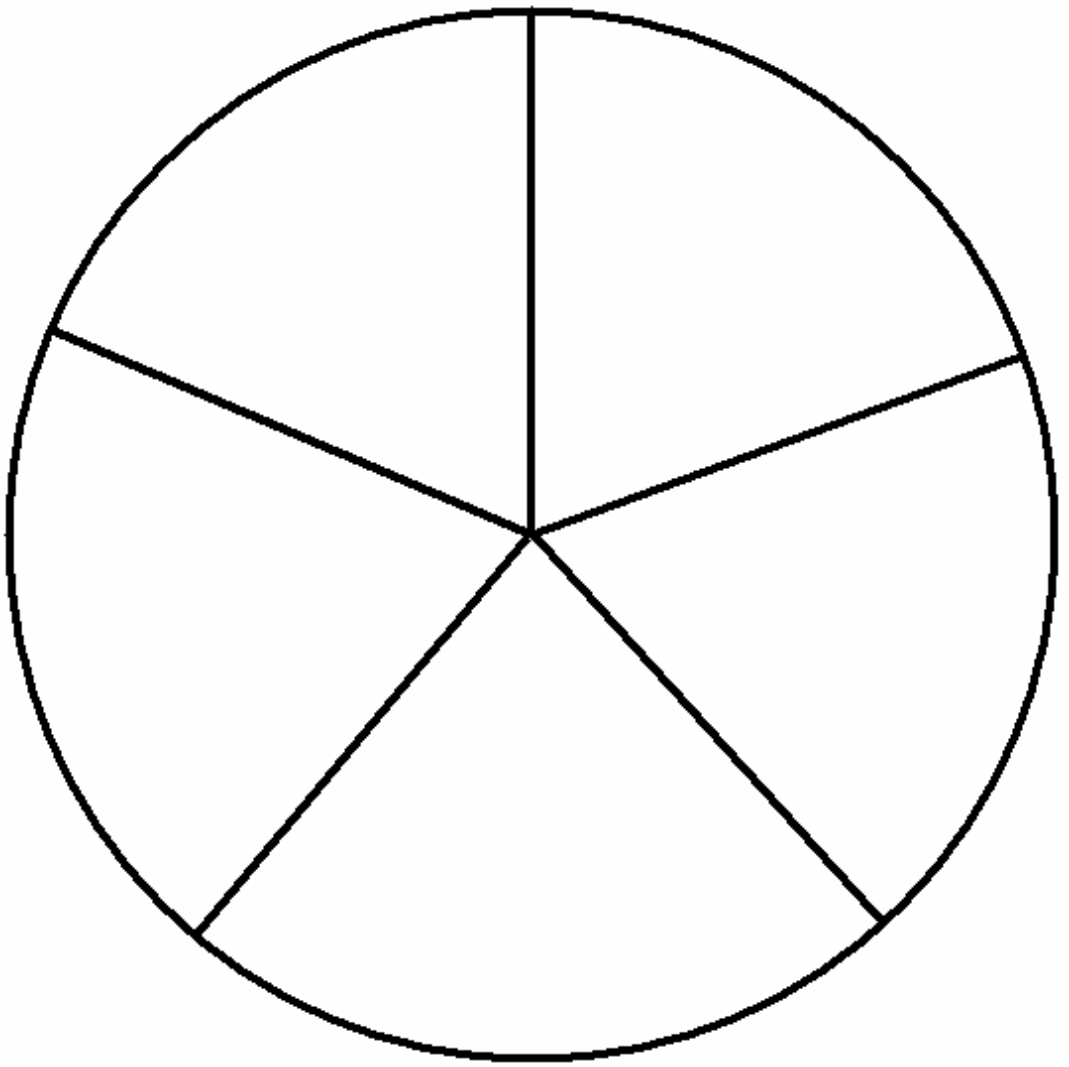

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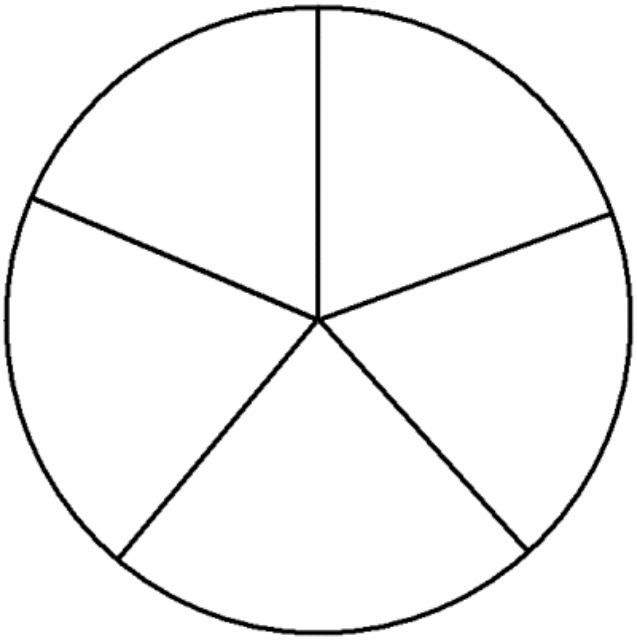

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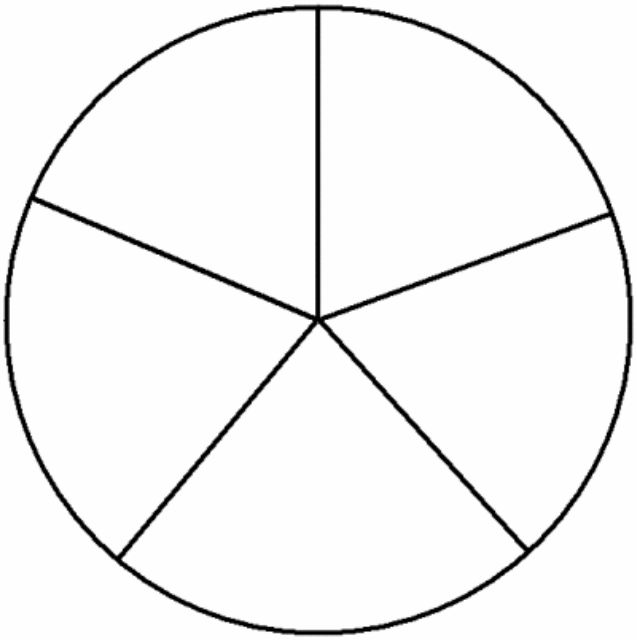

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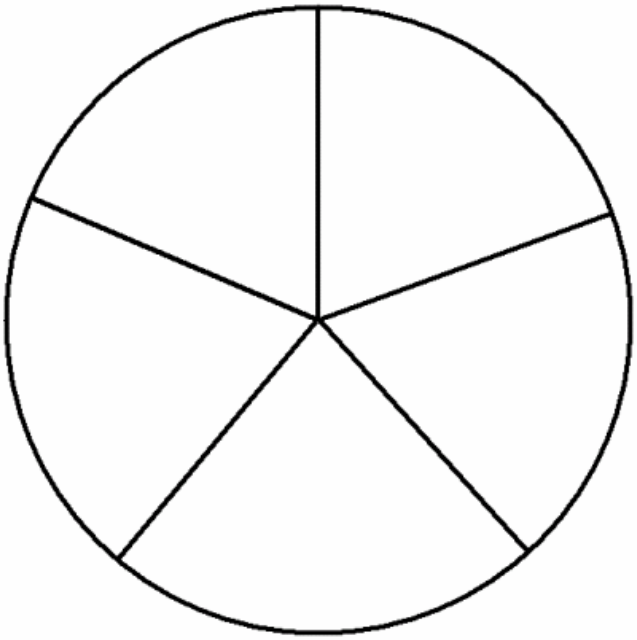

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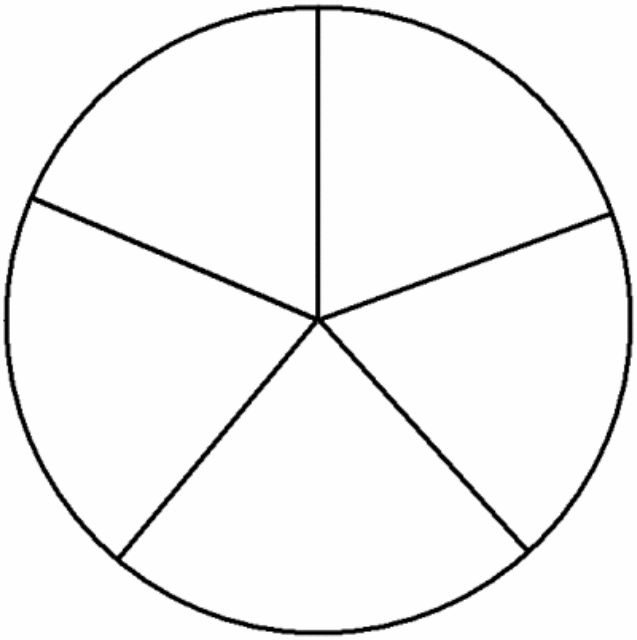

Concept Circles (Black Line)

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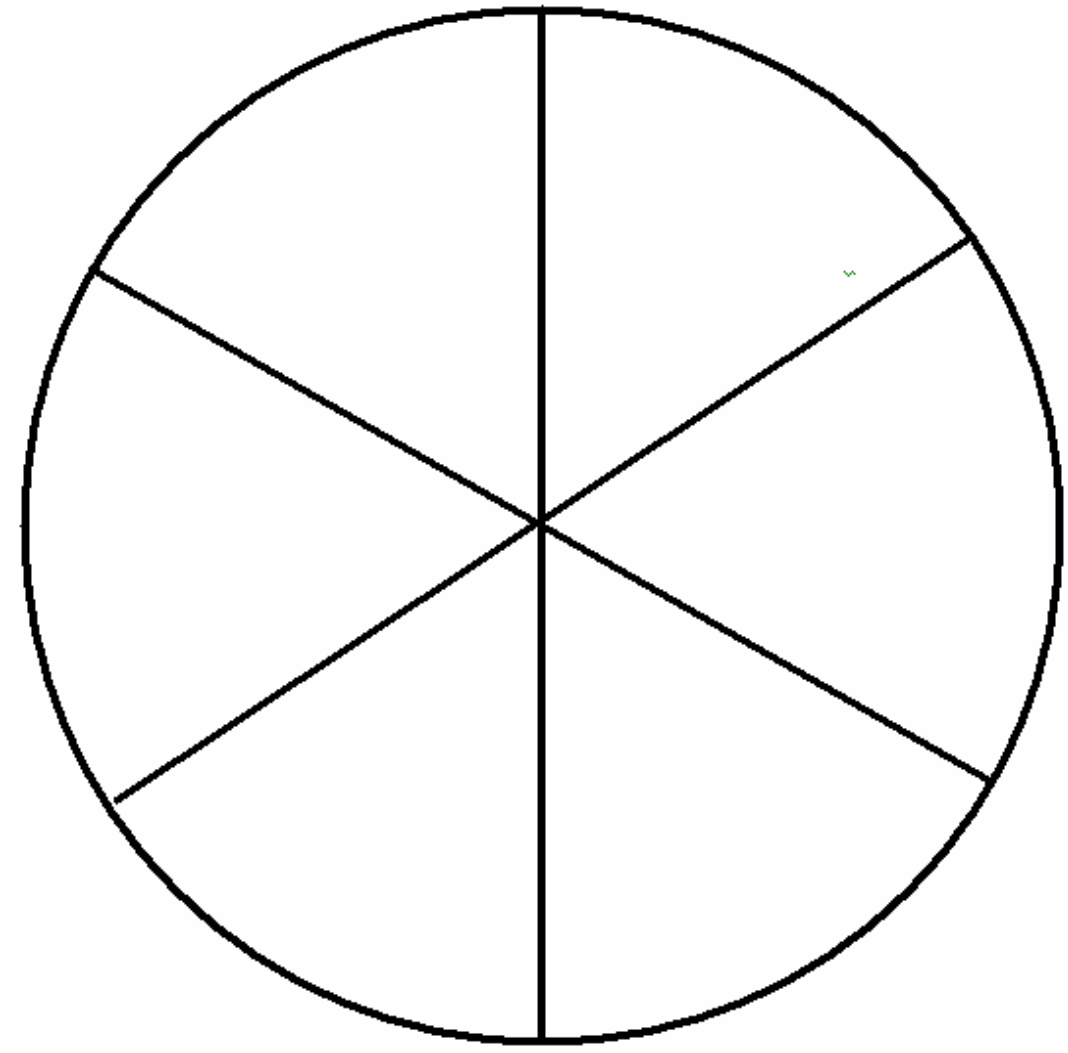

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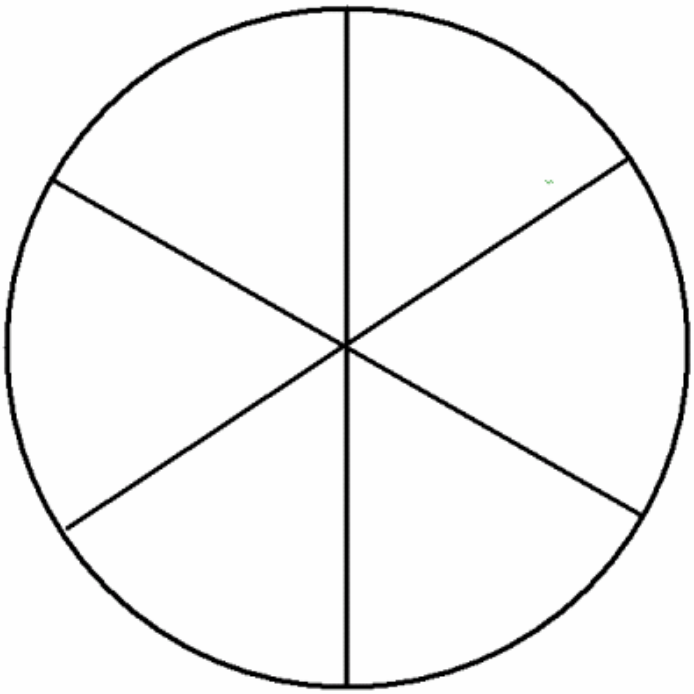

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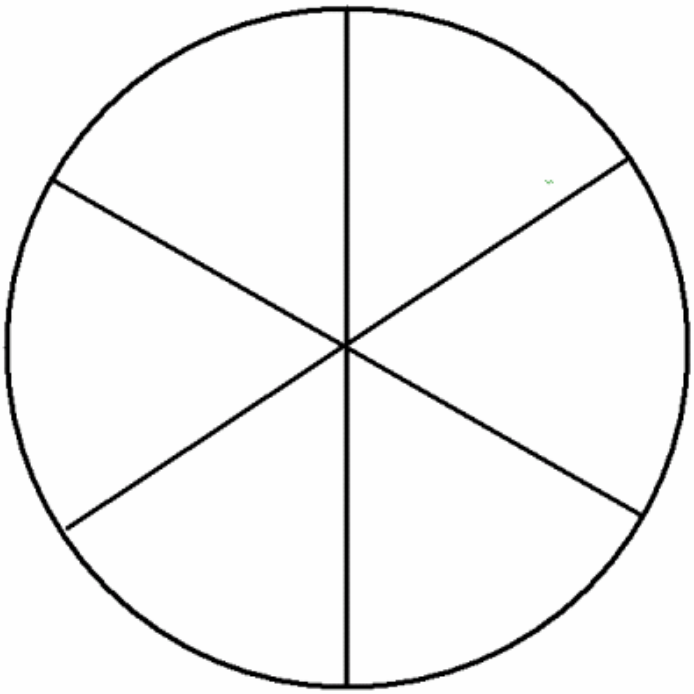

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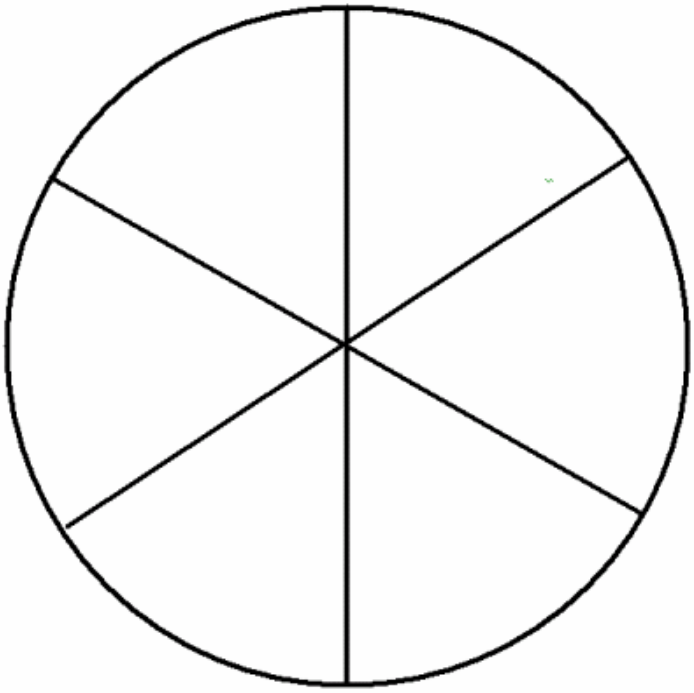

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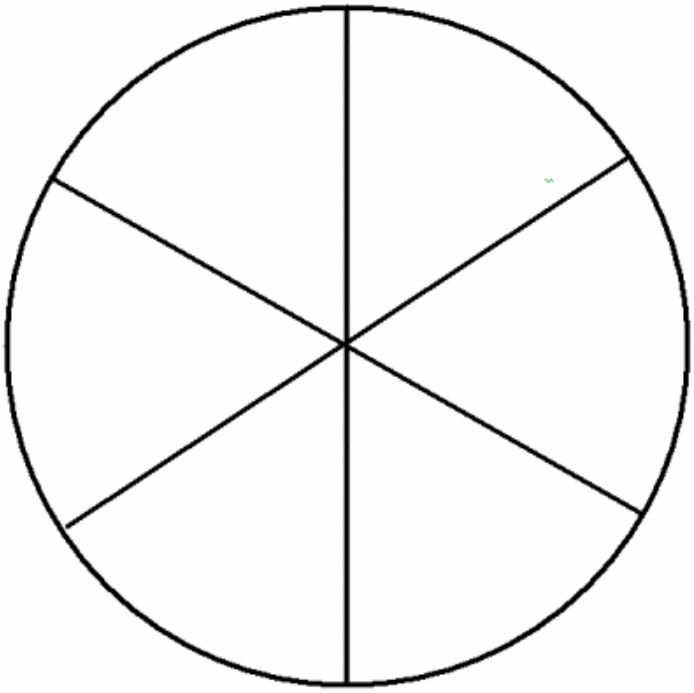
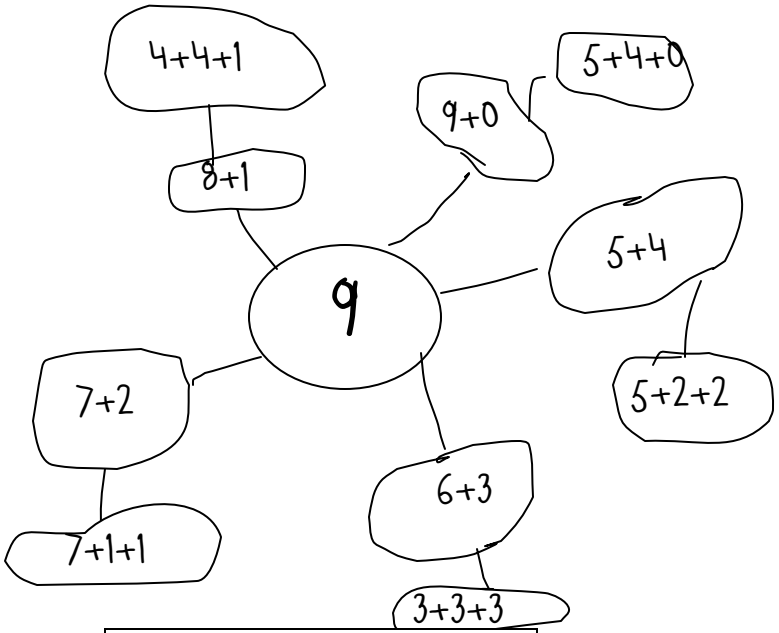


Word Web

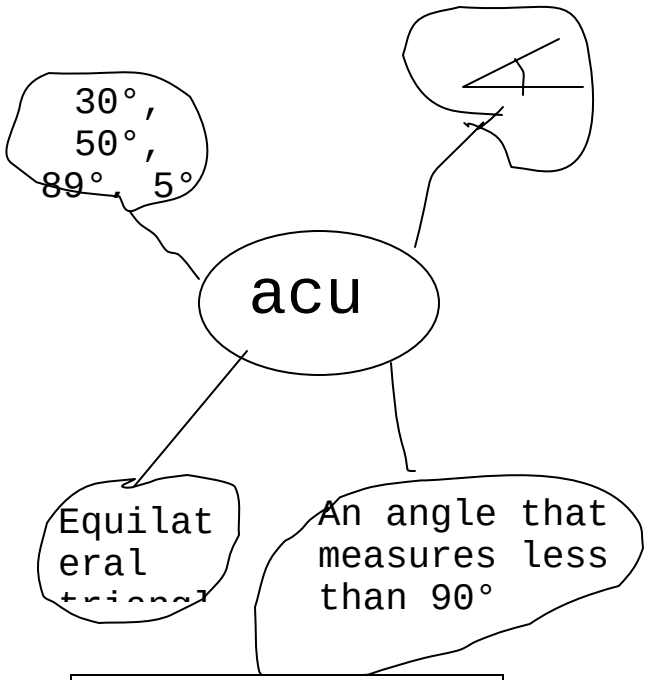
Word webs help students to visualize how characteristics, examples, and definitions relate and support a specific t vocabulary word or mathematical symbol.

How to use it:

1. Write the targeted vocabulary word inside a large circle. Make sure the circle is in the middle of the chart paper or overhead.
2. Dictate the students' related responses –e.g. definition, characteristic, or example— for the new vocabulary word outside of the middle circle.
3. Draw a circle, square, triangle, etc. around each response and connect it to the middle circle with a line.
4. DO NOT duplicate the black line master on page 20 for use as a whole group think aloud activity. Instead, construct the diagram over time with the students as their understanding and work around the related concept dictates. Use the black line master of this graphic organizer for independent student use and structured center activities.
5. Assign tasks that require the students to use this diagram as a strategy for taking notes and organizing their thinking around related concepts. This will prepare the students for note taking skills necessary in middle school.



1st grade student example



4th grade classroom example

Word Web (Black line)