

Check Digit Schemes

UPC Codes – Version A

Version A UPC codes have the form:

$$a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 a_9 a_{10} a_{11} a_{12}$$

The bar code one sees on the product represents the number in a form which the optical scanner can easily read. In a given store each UPC code is associated with a price. When the item is scanned at checkout the price is recorded for payment as well as many other statistics for store inventory and the assessment of what people like and don't like.

Here a_1 denotes the type of product as in the following table:

a_1	Product
0	General Groceries
2	Meat and Produce
3	Drugs and Health Products
4	Non-food items
5	Coupons
6,7	Other Items
1,8,9	Reserved for Future Use

$a_2 a_3 a_4 a_5 a_6$ identifies the manufacturer and $a_7 a_8 a_9 a_{10} a_{11}$ the product. a_{12} is the check digit and is chosen so that the following dot product relation is valid:

$$(3,1,3,1,3,1,3,1,3,1,3,1) \cdot (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}) \equiv 0 \pmod{10}$$

In other words,

$$3a_1 + a_2 + 3a_3 + a_4 + 3a_5 + a_6 + 3a_7 + a_8 + 3a_9 + a_{10} + 3a_{11} + a_{12} \equiv 0 \pmod{10}$$

Example:

The UPC code for my bottle of white-out is 0 41540 56301 3. Applying the check digit code:

$$3*0 + 1*4 + 3*1 + 1*5 + 3*4 + 1*0 + 3*5 + 1*6 + 3*3 + 1*0 + 3*1 + 1*3 = \underline{\hspace{2cm}}$$

The number obtained above is divisible by 10, thus there is a good chance that I typed UPC code correctly when putting it on this worksheet. However, there is still a chance that it was typed in wrong. Let's look at some of the possibilities.

The most common error in typing in a code might be to add or leave out a digit. That would obviously be caught immediately. The second most common error in typing in a code would be to get one of the numbers wrong. Pick a digit that's multiplied by 1 in the check digit formula (say the 6). Now, change the 6 to any other digit and check the dot product.

$$3*0 + 1*4 + 3*1 + 1*5 + 3*4 + 1*0 + 3*5 + 1*\underline{\hspace{1cm}} + 3*3 + 1*0 + 3*1 + 1*3 = \underline{\hspace{2cm}}$$

- ✓ Is there another number that will result in the dot product being 0 (mod 10)? _____
- ✓ Explain: _____

Now choose one of the numbers that's multiplied by a 3 in the check digit formula (say the second 4). Now, change the 4 to any other digit and check the dot product.

$$3*0 + 1*4 + 3*1 + 1*5 + 3* \underline{\quad} + 1*0 + 3*5 + 1*6 + 3*3 + 1*0 + 3*1 + 1*3 = \underline{\quad}$$

- ✓ Is there another number that will result in the dot product being 0 (mod 10)? _____

The next most common mistake would probably be to transpose two numbers. Suppose in typing the code the 5 and 6 near the middle were transposed. Calculate the new dot product.

$$3*0 + 1*4 + 3*1 + 1*5 + 3*4 + 1*0 + 3*\underline{6} + 1*\underline{5} + 3*3 + 1*0 + 3*1 + 1*3 = \underline{\quad}$$

Suppose instead that the 5 and 0 in the middle were transposed. Now calculate the dot product.

$$3*0 + 1*4 + 3*1 + 1*5 + 3*4 + 1*\underline{5} + 3*\underline{0} + 1*6 + 3*3 + 1*0 + 3*1 + 1*3 = \underline{\quad}$$

- ✓ Can you determine when transposing two numbers would not be detectable?

1) Obtain the UPC code from two different items available in the classroom and calculate the dot products using the check digit scheme.

Item _____

UPC Code _____ Dot Product _____

Item _____

UPC Code _____ Dot Product _____

Fill in the missing numbers from the following hypothetical UPC Codes:

2) 3 12345 67423 ____

3) 5 23408 5__435 2

4) How many different UPC Codes is it possible to create by filling in the two missing numbers in the code below? List all the combinations of numbers that would work.

2 343__1 034__2 5